BIOMEDE/ME 456: TISSUE MECHANICS
FALL 2018
Class Schedule: MW 8:30 AM - 10:00 AM, 1121 LBME

The course is divided into 2 parts:

**Part I:** Introduction to Continuum Mechanics relevant to Biological Tissues

Covers the fundamentals of continuum mechanics and constitutive modeling relevant for biological materials. Constitutive models covered include:

1. Linear Elasticity
2. Nonlinear Elasticity
3. Viscoelasticity
4. Optimization methods to fit mathematical models to collected mechanical testing data in MATLAB

**Part II:** Finite element modeling of tissue behavior using MIMICS/3-matics, HyperMesh, and COMSOL

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**Instructor:**
Rhima Coleman (rhimacol@umich.edu)
Assistant Professor
Department of Biomedical Engineering
Office: 2170 LBME

Office Hours in LBME 2185:
Mondays: 10-11:30am
Wednesdays: 10-11:30am
or send email to set up an appointment

**Grader:** Nicole Chen (nicchen@umich.edu)

GSI: Tiana Wong (tjwong@umich.edu)
Office Hours in LBME 2189:
Tuesdays and Thursdays: 1-3 pm.

**Texts**
Available online through the library:
- *Modelling Organs, Tissues, Cells and Devices Using MATLAB and COMSOL Multiphysics* — Socrates Dokos.
- *Cardiovascular Solid Mechanics Cells, Tissues, and Organs* — Humphrey, Jay D., author.
- *Continuum mechanics for engineers* — George Mase
- *A Concise Introduction to Linear Algebra* — Géza Schay
- *Tensor Algebra and Tensor Analysis for Engineers With Applications to Continuum Mechanics* — Itskov, Mikhail
- *Math refresher for scientists and engineers* — John R. Fanchi

Recommended study guides:
Schaum's Outline of Linear Algebra
Schaum’s Outline of Continuum Mechanics

Grading Criteria:
25% Homework (includes writing, MATLAB, and FEM assignments)
45% Biomechanics Project (written and oral components):
30% Midterm

General Course Policies:
Attendance at the lectures is not mandatory unless there is a guest speaker. Exams will include questions from the guest speaker and the absent student is expected to make arrangements with another student to obtain notes. Attendance and participation will be considered in assigning letter grades in borderline cases.

Students who must reschedule exams and or assignments due to religious observances or other personal matters should notify the instructors in advance. Students with disabilities who require special accommodations during classes or examinations should contact the Office of Services to Students with Disabilities to ensure that appropriate arrangements are made. The student is responsible for reminding the instructor of conflicts due to team activities and requirements for special accommodations as the need arises.

Assignments and examinations will be graded and returned to students as soon as possible after being handed in. Students should check the grading carefully. Any grade appeals must be submitted in writing within one week of the return of the assignment or exam.

Emails will be answered within 48hrs of receiving them. Proper email etiquette is expected for any communications with the instructor or GSI. Please include [BME456 or ME456] in the subject line of all emails.

All students in this class are bound by the College of Engineering Honor Code. You may not seek to gain an unfair advantage over your fellow students; you may not consult, look at, or possess the unpublished work of another without their permission; and you must appropriately acknowledge your use of another’s work. Any violation of the honor policies appropriate to each piece of course work will be reported to the Honor Council, and if guilt is established penalties may be imposed by the Honor Council and Faculty Committee on Discipline. Such penalties can include, but are not limited to, letter grade deductions or expulsion from the University. Collaboration policies on individual assignments will be described in the assignment handout. If you have any questions about the policies in this course, please consult the course instructor.

Topics to be covered (subject to change): Note that HW due dates may be adjusted based on the speed through the syllabus.

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<tr>
<th>Lect.</th>
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<th>Assignments</th>
<th>Lecture Topic</th>
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<tr>
<td>1</td>
<td>9/5</td>
<td>Class Overview/History of Continuum Mechanics</td>
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<td>Class Motivation</td>
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<tr>
<td>2</td>
<td>9/10</td>
<td>Top 3 tissue choices due</td>
<td>Mathematical Foundations/Indicial Notation</td>
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<td>3</td>
<td>9/12</td>
<td>Indicial Notation/Deformation and Strain</td>
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<td>1) Define the assumptions made while we are deriving the linear elastic equations (recall that these model idealized materials)</td>
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<td>2) Mapping between the undeformed deformed coordinate systems using the deformation tensor: $F_{ij}$</td>
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### 3) Derive the infinitesimal and finite strain tensors: \( \varepsilon_{ij}, E_{ij} \)

4) Left and right Cauchy strain tensors: \( B_{ij}, C_{ij} \)

### Stress for LE materials and NLE materials

1) Cauchy (true) stress: \( \sigma_{ij} \)

2) Equations of motion: \( \frac{\partial \sigma_{ij}}{\partial x} = 0 \)

3) Calculating surface stresses: \( t_i = \sigma_{ij} n_j \)

### Left and right Cauchy strain tensors:

\[
\begin{align*}
\varepsilon_{ij} &= \varepsilon_{ij}^{true} \\
\epsilon_{ij} &\equiv \frac{1}{2} (\nabla u : \nabla u) \\
B_{ij} &= \frac{1}{2} (\nabla u : \nabla u) \\
C_{ij} &= \frac{1}{2} (\nabla u : \nabla u) + \frac{1}{2} \delta_{ij} \nabla^2 u \\
\end{align*}
\]

### Stress for LE and NLE materials

#### Strain Energy Density (\( W \)) and energetic equality:

\[
\begin{align*}
\sigma_{ij} &= \frac{\partial W(\varepsilon_{ij})}{\partial \varepsilon_{ij}} \\
T_{ij} &= \frac{\partial W(F_{ij})}{\partial F_{ij}} \\
S_{ij} &= \frac{\partial W(E_{ij})}{\partial E_{ij}} = 2 \left[ \frac{\partial W(C_{ij})}{\partial C_{ij}} \right] \\
\end{align*}
\]

#### LE constitutive model/Mechanical Isotropy

Cauchy stress: \( \sigma_{ij} = P/A = C_{ijkl} \varepsilon_{kl} \)

1) 1st PK: \( T_{ij} = P/A' = J \sigma_{ij} F_{ij}^{-1} \)

2) 2nd PK: \( S_{ij} = P'/A' = J F_{ij}^{-1} \sigma_{ij} F_{ij}^{-1} \)

### NLE constitutive model/Assumptions of Incompressibility

1) \( \text{SED as a function of invariants} \)

2) Specific tissue models: Ogden, Mooney-Rivlin, Neo-Hookean

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**HW#1 due**

Stress for LE materials and NLE materials

Draft of mechanical testing protocol due

Stress for LE and NLE materials

NLE constitutive model/Viscoelasticity

Final mechanical testing protocol due

Fitting Non-Linear Elasticity Models in MATLAB

HW#3 due

Midterm Review